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Effect of damping on dynamic behavior of a piezothermoelastic laminate considering the effects of transverse shear

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Abstract

In this paper, we analyze dynamic behavior of a piezothermoelastic laminate considering the effect of damping due to interlaminar shear and the effect of transverse shear. The analytical model is a rectangular laminate composed of fiber-reinforced laminae and piezoelectric layers. The model is assumed to be a symmetric cross-ply laminate with all egdes simply supported and to be subjected to mechanical, thermal and electrical loads varying arbitrarily with time. Behavior of the laminate is analyzed based on the first-order shear deformation theory. The effect of damping due to interlaminar shear is incorporated into our analysis by introducing the interlaminar shear stresses which satisfy the Newton's law of viscosity. Solutions of the following quantities are obtained: (1) natural frequencies of the laminate, (2) weight functions for the deflection and rotations and (3) unsteady deflection due to loads varying arbitrarily with time. Moreover, numerical examples of the solutions are shown to examine the effects of damping and transverse shear on dynamic behavior of the laminate and how the voltage applied to the laminate decreases the deflection due to mechanical or thermal loads.

Keywords: Damping; Piezothermoelasticity; Laminate; Thermal stress; Transverse shear

1. Introduction

Recently smart structures have attracted much attention in engineering, medicine and other fields. Piezoelectric materials are used as an important element of smart structures and are often attached to structural laminates such as graphite/epoxy. The laminates composed of them have been used as devices for vibration-control, shape-control and so forth in adaptive structures (Tzou and Anderson, 1992) and have become to be used under severe mechanical and thermal environment. The laminate, which is called *piezothermoelastic laminate*, was studied by many authors for *static* behavior with the aim of shape-control (Wu and Tauchert, 1980a,b; Tauchert, 1992; Noda and Kimura, 1998; Ishihara and Noda, 2000).

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During operation such as vibration-control, piezothermoelastic laminates are often subjected to dynamically changing mechanical and thermal loads unavoidably and are also subjected to electrical load to compensate the effect of those unavoidable loads. Therefore, *dynamic behavior* of piezothermoelastic laminates has been analyzed by several authors. Shen and Kuang (1999) analyzed *steady* vibration of a piezothermoelastic laminate due to harmonic excitation taking into account the effect of transverse shear. In our previous paper (Ishihara and Noda, 2002), we studied *transient* dynamic behavior of a piezothermoelastic laminate considering the effect of transverse shear under the first-order shear deformation theory (FSDT).

Dynamic analysis aforementioned (Ishihara and Noda, 2002) does not take into account the effect of damping while a laminate generally undergoes the effect. Damping changes such dynamic characters as the natural frequency, transient behavior and so forth. In particular, the natural frequency is underestimated by the analysis without regard to the effect of damping. Therefore, the analysis taking damping into account is important to estimate dynamic characters properly. Actually, Tang and Xu (1995) took damping into account and obtained the response of a piezothermoelastic laminate based on the classical laminate theory (CLT). In their analysis (Tang and Xu, 1995), they introduced damping terms proportional to *translation* velocity into equations of motion of the laminate. Damping in laminates is considered to be owing mainly to the existence of interlaminar layers which are infinitesimal in thickness but finite in damping effect. Therefore, when the effect of the damping on dynamic behavior of piezothermoelastic laminates is considered, it is more important to discuss the damping due to interlaminar shear than that proportional to *translation* velocity.

Therefore, we study the effect of damping due to interlaminar shear on dynamic behavior of a piezothermoelastic laminate taking into account the effect of transverse shear assuming the existence of infinitesimal interlaminar layers. The analytical model is a rectangular laminate composed of fiber-reinforced laminae and piezoelectric layers. The model is assumed to be a symmetric cross-ply laminate with all egdes simply supported. The laminate is unavoidably subjected to mechanical and thermal loads which are dynamically changing and is subjected to electrical loads to compensate the effect of those unavoidable loads.

Behavior of the laminate is analyzed based on the FSDT. The effect of damping due to interlaminar shear is incorporated into our analysis by introducing the interlaminar shear stresses which satisfy the Newton's law of viscosity, that is, the shear stresses which are proportional to velocity gradients in the infinitesimal interlaminar layers. The equations of motion are expressed by deflection of the laminate and rotations of the cross-sections. The deflection and rotations are expressed by double Fourier series. Laplace transform is introduced to the equations of motion. As a result, the following quantities are obtained: (1) natural frequencies of the laminate, (2) weight functions for the deflection and rotations and (3) transient deflection due to loads varying dynamically with time.

Moreover, numerical examples of the solutions are shown to examine the effect of damping due to interlaminar shear and the effect of transverse shear on dynamic behavior of the laminate and how the voltage applied to the laminate decreases the deflection due to mechanical or thermal loads.

2. Analysis

2.1. Problem

The analytical model is shown in Fig. 1. The model is a rectangular laminate with dimension $a \times b \times h$ composed of N layers: two of N layers ($z_{k-1} \leqslant z \leqslant z_k$, $z_{k'-1} \leqslant z \leqslant z_{k'}$) exhibit piezoelectricity while other layers do not. The laminate is a cross-ply laminate: all the layers exhibit 2 mm symmetry with respect to xy-plane and the principal axes of anisotropy coincide with the axes of the Cartesian coordinate system (x, y, z). The

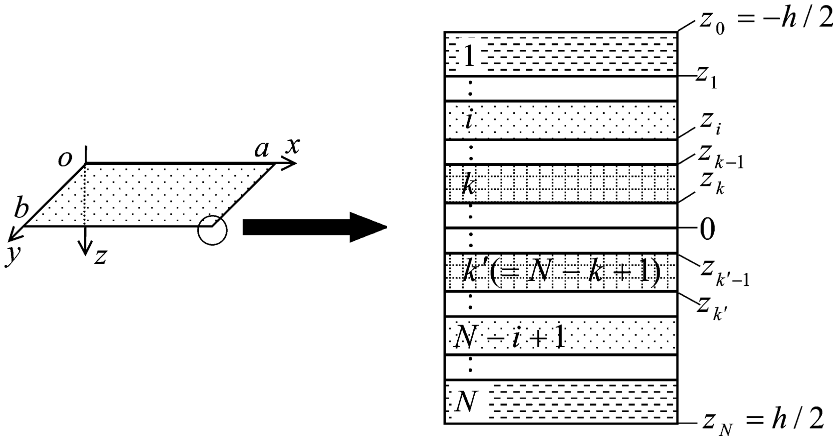

Fig. 1. Analytical model.

layers are laminated symmetrically with respect to the central plane $z = 0$: the ith and $(N - i + 1)$th layers are composed of the same material and have the same anisotropy with respect to the Cartesian coordinate system (x, y, z). All edges of the laminate are simply supported.

The laminate is subjected to *unsteady* loads varying arbitrarily with time t: transverse load $q(x, y, t)$; temperature distribution $T_0(x, y, t)$ and $T_N(x, y, t)$ on the upper surface $(z = -h/2)$ and the lower surface $(z = h/2)$, respectively; electric potential $V^k(x, y, t)$ and $V^{k'}(x, y, t)$ on the upper surface $(z = z_{k-1})$ of the kth layer and the lower surface $(z = z_{k'})$ of the k'th layer, respectively. The lower surface $(z = z_k)$ of the kth layer and the upper surface $(z = z_{k'-1})$ of the k'th layer are both the level surfaces of electric potential.

2.2. Analytical procedure

In order to take into account the effect of transverse shear, the analytical procedure is based on the FSDT. The displacement components in x-, y- and z-directions are taken to be, respectively,

$$u = u^0 + z\psi_x, \quad v = v^0 + z\psi_y, \quad w = w^0, \tag{1}$$

where the superscript 0 denotes the quantities at the central plane; and ψ_x and ψ_y denote rotations of the cross-sections perpendicular to x- and y-axes, respectively. In order to take into account the effect of damping due to shear, we introduce the external force (per unit area) τ_{ij} which acts on i-plane in j-direction $(i, j = x, y, z;\ i \neq j)$ and satisfies the Newton's law of viscosity as

$$\tau_{ij} = \mu_{ij} \frac{\partial}{\partial i}\left(\frac{\partial u_j}{\partial t}\right), \tag{2}$$

where u_x, u_y and u_z denote u, v and w, respectively, and μ_{ij} is referred to as viscosity.

We assume for the present that external force τ_{ij}, hence viscosity μ_{ij}, is distributed throughout the laminate. Then, referring to Fig. 2, the equations of motion incorporating the effect of damping due to shear are derived as

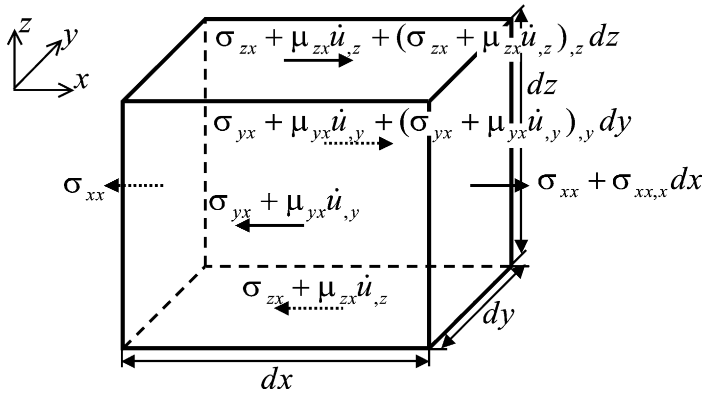

Fig. 2. Equilibrium of stresses and external forces in x-direction.

$$\left.\begin{aligned} &\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{yx}}{\partial y}+\frac{\partial\sigma_{zx}}{\partial z}+\frac{\partial}{\partial y}\left[\mu_{yx}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t}\right)\right]+\frac{\partial}{\partial z}\left[\mu_{zx}\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial t}\right)\right]=\rho\frac{\partial^2 u}{\partial t^2}\\ &\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{zy}}{\partial z}+\frac{\partial}{\partial z}\left[\mu_{zy}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial t}\right)\right]+\frac{\partial}{\partial x}\left[\mu_{xy}\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial t}\right)\right]=\rho\frac{\partial^2 v}{\partial t^2}\\ &\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\partial}{\partial x}\left[\mu_{xz}\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial t}\right)\right]+\frac{\partial}{\partial y}\left[\mu_{yz}\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial t}\right)\right]=\rho\frac{\partial^2 w}{\partial t^2} \end{aligned}\right\}, \tag{3}$$

where σ_{ij} and ρ denote stresses and mass density, respectively.

Moreover, we assume that the external force τ_{ij}, hence viscosity μ_{ij}, occurs only in the infinitesimal interlaminar layers described by $z = z_i$ $(i = 1, \ldots, N-1)$ as a consequence of the limit process where the thickness of each interlaminar layer tends to zero. Then, viscosities can be expressed as

$$\mu_{zx}=\mu_{zy}\equiv\sum_{i=1}^{N-1}\delta(z-z_i)\mu_i,\quad \mu_{xy}=\mu_{xz}=\mu_{yx}=\mu_{yz}=0, \tag{4}$$

where $\delta(\cdot)$ denotes the Dirac's delta function and μ_i denotes the viscosity in infinitesimal interlaminar layer z_i. Substitution of Eqs. (1) and (4) into Eq. (3); integration of the third equation of Eq. (3) for $-h/2 \leqslant z \leqslant h/2$; integration of the first and second equations of Eq. (3) for $-h/2 \leqslant z \leqslant h/2$ with z multiplied give equations of motion for the laminate as follows:

$$\left.\begin{aligned} &\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+q=P\frac{\partial^2 w}{\partial t^2}\\ &\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=I\frac{\partial^2\psi_x}{\partial t^2}+\mu\frac{\partial\psi_x}{\partial t},\quad \frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y=I\frac{\partial^2\psi_y}{\partial t^2}+\mu\frac{\partial\psi_y}{\partial t} \end{aligned}\right\}, \tag{5}$$

where the definitions of Q_x, Q_y, M_x, M_y, M_{xy}, P, I and μ are given by

$$\{Q_x,Q_y\}=\int_{-h/2}^{h/2}\{\sigma_{xz},\sigma_{yz}\}\,\mathrm{d}z,\quad \{M_x,M_y,M_{xy}\}=\int_{-h/2}^{h/2}\{\sigma_{xx},\sigma_{yy},\sigma_{xy}\}z\,\mathrm{d}z, \tag{6}$$

$$\{P,I\}=\int_{-h/2}^{h/2}\{1,z^2\}\rho\,\mathrm{d}z,\quad \mu=\sum_{i=1}^{N-1}\mu_i. \tag{7}$$

Here, P and I are the coefficients of translation and rotary inertia, respectively. Hereafter, we refer to μ as the damping coefficient of the laminate. The constitutive equations of piezothermoelasticity for the symmetrical cross-ply laminate are

$$\left.\begin{aligned}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix} &= \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & 0\\ \overline{Q}_{12} & \overline{Q}_{22} & 0\\ 0 & 0 & \overline{Q}_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} - \begin{bmatrix}0 & 0 & \bar{e}_{31}\\ 0 & 0 & \bar{e}_{32}\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix} - \begin{Bmatrix}\bar{\lambda}_1\\ \bar{\lambda}_2\\ 0\end{Bmatrix}T\\ \begin{Bmatrix}\sigma_{yz}\\ \sigma_{xz}\end{Bmatrix} &= \begin{bmatrix}\overline{Q}_{44} & 0\\ 0 & \overline{Q}_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} - \begin{bmatrix}0 & \bar{e}_{24}\\ \bar{e}_{15} & 0\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\end{Bmatrix}\end{aligned}\right\}, \tag{8}$$

where ε_{ij} and γ_{ij} denote strains, E_i denotes electric fields, T denotes temperature, $\overline{Q}_{ij}$, $\bar{e}_{ij}$ and $\bar{\lambda}_i$ denote elastic stiffness coefficients, piezoelectric coefficients and stress–temperature coefficients all of which are reduced and transformed (Jonnalagadda et al., 1994). From Eq. (1), strains are expressed as

$$\left.\begin{aligned}&\varepsilon_{xx} = \frac{\partial u^0}{\partial x} + z\frac{\partial \psi_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial v^0}{\partial y} + z\frac{\partial \psi_y}{\partial y}, \quad \gamma_{xy} = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + z\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right)\\ &\gamma_{xz} = \frac{\partial w}{\partial x} + \psi_x, \quad \gamma_{yz} = \frac{\partial w}{\partial y} + \psi_y\end{aligned}\right\}. \tag{9}$$

Electric fields are expressed by electric potential Φ as

$$E_x = -\frac{\partial \Phi}{\partial x}, \quad E_y = -\frac{\partial \Phi}{\partial y}, \quad E_z = -\frac{\partial \Phi}{\partial z}. \tag{10}$$

Substitution of Eq. (9) into Eq. (8); integration of the first equation of Eq. (8) for $-h/2 \leqslant z \leqslant h/2$ with z multiplied; integration of the second equation of Eq. (8) for $-h/2 \leqslant z \leqslant h/2$ give

$$\begin{aligned}\begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix} &= \begin{bmatrix}D_{11} & D_{12} & 0\\ D_{12} & D_{22} & 0\\ 0 & 0 & D_{66}\end{bmatrix}\begin{Bmatrix}\dfrac{\partial \psi_x}{\partial x}\\ \dfrac{\partial \psi_y}{\partial y}\\ \dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x}\end{Bmatrix} - \begin{Bmatrix}M_x^T + M_x^E\\ M_y^T + M_y^E\\ 0\end{Bmatrix},\\ \begin{Bmatrix}Q_y\\ Q_x\end{Bmatrix} &= \begin{bmatrix}S_{44} & 0\\ 0 & S_{55}\end{bmatrix}\begin{Bmatrix}\dfrac{\partial w}{\partial y} + \psi_y\\ \dfrac{\partial w}{\partial x} + \psi_x\end{Bmatrix} - \begin{Bmatrix}Q_y^E\\ Q_x^E\end{Bmatrix},\end{aligned} \tag{11}$$

where the definitions of D_{ij}, S_{ij}, M_x^T, M_y^T, M_x^E, M_y^E, Q_x^E and Q_y^E are given as

$$D_{ij} = \int_{-h/2}^{h/2} \overline{Q}_{ij} z^2\,\mathrm{d}z \quad (i,j = 1,2,6), \quad \{S_{44}, S_{55}\} = \int_{-h/2}^{h/2} \{k_2^2 \overline{Q}_{44}, k_1^2 \overline{Q}_{55}\}\mathrm{d}z, \tag{12}$$

$$\begin{Bmatrix}M_x^T\\ M_y^T\end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix}\bar{\lambda}_1\\ \bar{\lambda}_2\end{Bmatrix} Tz\,\mathrm{d}z, \quad \begin{Bmatrix}M_x^E\\ M_y^E\end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix}\bar{e}_{31}\\ \bar{e}_{32}\end{Bmatrix} E_z z\,\mathrm{d}z, \quad \begin{Bmatrix}Q_x^E\\ Q_y^E\end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix}\bar{e}_{15}E_x\\ \bar{e}_{24}E_y\end{Bmatrix}\mathrm{d}z, \tag{13}$$

where parameters k_1 and k_2 in Eq. (12) are introduced to take into account non-uniform shear strain distribution through the plate thickness. By substituting Eq. (11) into Eq. (5), the equations of motion are expressed by deflection and rotations as follows:

$$\left.\begin{aligned}
&S_{55}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \psi_x}{\partial x}\right)+S_{44}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial \psi_y}{\partial y}\right)-\left(\frac{\partial Q_x^E}{\partial x}+\frac{\partial Q_y^E}{\partial y}\right)+q=P\frac{\partial^2 w}{\partial t^2}\\
&D_{11}\frac{\partial^2\psi_x}{\partial x^2}+D_{66}\frac{\partial^2\psi_x}{\partial y^2}+(D_{12}+D_{66})\frac{\partial^2\psi_y}{\partial x\,\partial y}-S_{55}\left(\frac{\partial w}{\partial x}+\psi_x\right)+Q_x^E-\frac{\partial(M_x^T+M_x^E)}{\partial x}=I\frac{\partial^2\psi_x}{\partial t^2}+\mu\frac{\partial\psi_x}{\partial t}\\
&(D_{12}+D_{66})\frac{\partial^2\psi_x}{\partial x\,\partial y}+D_{66}\frac{\partial^2\psi_y}{\partial x^2}+D_{22}\frac{\partial^2\psi_y}{\partial y^2}-S_{44}\left(\frac{\partial w}{\partial y}+\psi_y\right)+Q_y^E-\frac{\partial(M_y^T+M_y^E)}{\partial y}=I\frac{\partial^2\psi_y}{\partial t^2}+\mu\frac{\partial\psi_y}{\partial t}
\end{aligned}\right\}. \tag{14}$$

As the laminate is simply-supported at all edges, we have

$$\left.\begin{aligned}
&x=0,a;\quad w=0,\quad M_x=0,\quad \psi_y=0\\
&y=0,b;\quad w=0,\quad M_y=0,\quad \psi_x=0
\end{aligned}\right\}. \tag{15}$$

We assume that the laminate is sufficiently thin, therefore, that the distributions of temperature and electric potential are linear with respect to thickness direction as

$$T(x,y,z,t)=\frac{1}{2}[T_N(x,y,t)+T_0(x,y,t)]+\frac{z}{h}[T_N(x,y,t)-T_0(x,y,t)]\quad(-h/2\leqslant z\leqslant h/2), \tag{16}$$

$$\left.\begin{aligned}
&\Phi(x,y,z,t)=V^k(x,y,t)\frac{z_k-z}{z_k-z_{k-1}}\quad(z_{k-1}\leqslant z\leqslant z_k)\\
&\Phi(x,y,z,t)=V^{k'}(x,y,t)\frac{z-z_{k'-1}}{z_{k'}-z_{k'-1}}\quad(z_{k'-1}\leqslant z\leqslant z_{k'})
\end{aligned}\right\}. \tag{17}$$

Moreover, we assume that $q(x,y,t)$, $T_0(x,y,t)$, $T_N(x,y,t)$, $V^k(x,y,t)$ and $V^{k'}(x,y,t)$ are expressed by double Fourier series as follows:

$$q(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}(t)\sin\alpha_m x\sin\beta_n y, \tag{18}$$

$$\{T_0(x,y,t),T_N(x,y,t)\}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{T_{0,mn}(t),T_{N,mn}(t)\}\sin\alpha_m x\sin\beta_n y, \tag{19}$$

$$\left\{V^k(x,y,t),V^{k'}(x,y,t)\right\}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{V_{mn}^k(t),V_{mn}^{k'}(t)\right\}\sin\alpha_m x\sin\beta_n y, \tag{20}$$

where

$$\alpha_m=\frac{m\pi}{a},\quad \beta_n=\frac{n\pi}{b}. \tag{21}$$

In order to satisfy Eq. (15), we assume that deflection and rotations are also expressed by double Fourier series as follows:

$$\left.\begin{aligned}
&w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}w_{mn}(t)\sin\alpha_m x\sin\beta_n y\\
&\psi_x=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\psi_{x,mn}(t)\cos\alpha_m x\sin\beta_n y,\quad \psi_y=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\psi_{y,mn}(t)\sin\alpha_m x\cos\beta_n y
\end{aligned}\right\}. \tag{22}$$

Then, the equations of motion expressed by the Fourier coefficients of deflection and rotaions are obtained. By substituting Eqs. (10), (13), (16)–(22) into Eq. (14), we have

$$\mathbf{M}\ddot{\mathbf{x}}_{mn}(t) + \mathbf{C}\dot{\mathbf{x}}_{mn}(t) + \mathbf{K}_{mn}\mathbf{x}_{mn}(t) = \mathbf{p}_{mn}(t), \tag{23}$$

where

$$\left.\begin{aligned}
&\mathbf{x}_{mn}(t) = \{w_{mn}(t)/h, \psi_{x,mn}(t), \psi_{y,mn}(t)\}^{\mathrm{T}} \\
&\mathbf{M} = \begin{bmatrix} Ph & 0 & 0 \\ & I/h & 0 \\ \text{sym.} & & I/h \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 \\ & \mu/h & 0 \\ \text{sym.} & & \mu/h \end{bmatrix}, \quad \mathbf{K}_{mn} = \begin{bmatrix} k_{11,mn} & k_{12,mn} & k_{13,mn} \\ & k_{22,mn} & k_{23,mn} \\ \text{sym.} & & k_{33,mn} \end{bmatrix} \\
&k_{11,mn} = (S_{55}\alpha_m^2 + S_{44}\beta_n^2)h, \quad k_{12,mn} = S_{55}\alpha_m, \quad k_{13,mn} = S_{44}\beta_n \\
&k_{22,mn} = d_{11,mn}/h, \quad k_{33,mn} = d_{22,mn}/h, \quad k_{23,mn} = d_{12,mn}/h \\
&d_{11,mn} = D_{11}\alpha_m^2 + D_{66}\beta_n^2 + S_{55}, \quad d_{22,mn} = D_{66}\alpha_m^2 + D_{22}\beta_n^2 + S_{44}, \quad d_{12,mn} = (D_{12} + D_{66})\alpha_m\beta_n \\
&\mathbf{p}_{mn}(t) = q_{mn}(t)\cdot\{1,0,0\}^{\mathrm{T}} - \{0, \alpha_m H_{mn}(t)/h, \beta_n I_{mn}(t)/h\}^{\mathrm{T}} \\
&\qquad -(z_k + z_{k-1})/(2h)\cdot[V_{mn}^k(t) + V_{mn}^{k'}(t)]\cdot\{0, \bar{e}_{31}\alpha_m, \bar{e}_{32}\beta_n\}^{\mathrm{T}} \\
&\qquad -(z_k - z_{k-1})/(2h)\cdot[V_{mn}^k(t) + V_{mn}^{k'}(t)]\cdot\left\{(\bar{e}_{15}\alpha_m^2 + \bar{e}_{24}\beta_n^2)h, \bar{e}_{15}\alpha_m, \bar{e}_{24}\beta_n\right\}^{\mathrm{T}} \\
&\{H_{mn}(t), I_{mn}(t)\} = \sum_{i=1}^{N}(z_i^3 - z_{i-1}^3)/(3h)[T_{N,mn}(t) - T_{0,mn}(t)]\cdot\{\bar{\lambda}_1, \bar{\lambda}_2\}
\end{aligned}\right\}. \tag{24}$$

Eq. (23) is solved by using the Laplace transform (Sneddon, 1972). By applying the transform for variable t to Eq. (23), we have

$$\mathbf{x}_{mn}^*(s) = (s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}_{mn})^{-1}\cdot\{\mathbf{p}_{mn}^*(s) + s\mathbf{M}\mathbf{x}_{mn}(0) + [\mathbf{M}\dot{\mathbf{x}}_{mn}(0) + \mathbf{C}\mathbf{x}_{mn}(0)]\}, \tag{25}$$

where the functions with superscript $*$ denote the Laplace transform of the functions without superscript $*$ and s denotes the parameter of the Laplace transform. $(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}_{mn})^{-1}$, which means the transfer function, has different expressions for the case with rotary inertia taken into account ($I \neq 0$) and for the case with rotary inertia disregarded ($I = 0$). For the former case ($I \neq 0$), $(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}_{mn})^{-1}$ is expressed as

$$(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}_{mn})^{-1} = \frac{1}{(Ph)(I/h)^2\prod_{i=1}^{3}[(s+\gamma_{i,mn}\omega_{i,mn})^2 + (1-\gamma_{i,mn}^2)\omega_{i,mn}^2]}\begin{bmatrix} D_{mn,11}(s) & D_{mn,12}(s) & D_{mn,13}(s) \\ & D_{mn,22}(s) & D_{mn,23}(s) \\ \text{sym.} & & D_{mn,33}(s) \end{bmatrix}, \tag{26}$$

where $D_{mn,ij}(s)$ is given as

$$\left.\begin{aligned}
&D_{mn,ij}(s) = D_{0,mn,ij}(s) + D_{\mu,mn,ij}(s) \\
&D_{0,mn,11}(s) = (I/h)^2 s^4 + (I/h)(k_{22,mn} + k_{33,mn})s^2 + (k_{22,mn}k_{33,mn} - k_{23,mn}^2) \\
&D_{0,mn,22}(s) = (Ph)(I/h)s^4 + [(Ph)k_{33,mn} + (I/h)k_{11,mn}]s^2 + (k_{11,mn}k_{33,mn} - k_{13,mn}^2) \\
&D_{0,mn,33}(s) = (Ph)(I/h)s^4 + [(Ph)k_{22,mn} + (I/h)k_{11,mn}]s^2 + (k_{11,mn}k_{22,mn} - k_{12,mn}^2) \\
&D_{0,mn,12}(s) = -[(I/h)k_{12,mn}s^2 + (k_{12,mn}k_{33,mn} - k_{13,mn}k_{23,mn})] \\
&D_{0,mn,13}(s) = -[(I/h)k_{13,mn}s^2 + (k_{13,mn}k_{22,mn} - k_{12,mn}k_{23,mn})] \\
&D_{0,mn,23}(s) = -[(Ph)k_{23,mn}s^2 + (k_{23,mn}k_{11,mn} - k_{12,mn}k_{13,mn})] \\
&D_{\mu,mn,11}(s) = (\mu/h)s[2(I/h)s^2 + (\mu/h)s + (k_{22,mn} + k_{33,mn})] \\
&D_{\mu,mn,22}(s) = D_{\mu,mn,33}(s) = (\mu/h)s[(Ph)s^2 + k_{11,mn}] \\
&D_{\mu,mn,12}(s) = -(\mu/h)k_{12,mn}s, \quad D_{\mu,mn,13}(s) = -(\mu/h)k_{13,mn}s, \quad D_{\mu,mn,23}(s) = 0
\end{aligned}\right\} \tag{27}$$

and $\omega_{i,mn}$ and $\gamma_{i,mn}$ ($i = 1, 2, 3$) denote the undamped natural frequencies and the damping factors, respectively, which are determined by the iterative method to solve the following two sets of equations:

$$\left\{\omega_{1,mn}^2, \omega_{2,mn}^2, \omega_{3,mn}^2\right\} = \frac{1}{3}a_{\omega 2,mn}\cdot\{1,1,1\} - 2\sqrt{\frac{q_{\omega,mn}}{3}}\cdot\left\{\cos\left(\frac{\theta_{\omega,mn}}{3}\right), \cos\left(\frac{\theta_{\omega,mn}+2\pi}{3}\right), \cos\left(\frac{\theta_{\omega,mn}-2\pi}{3}\right)\right\} \tag{28}$$

and

$$\begin{bmatrix} 1 & 1 & 1 \\ \omega_{2,mn}^2+\omega_{3,mn}^2 & \omega_{3,mn}^2+\omega_{1,mn}^2 & \omega_{1,mn}^2+\omega_{2,mn}^2 \\ \omega_{2,mn}^2\omega_{3,mn}^2 & \omega_{3,mn}^2\omega_{1,mn}^2 & \omega_{1,mn}^2\omega_{2,mn}^2 \end{bmatrix}\begin{Bmatrix} \gamma_{1,mn}\omega_{1,mn} \\ \gamma_{2,mn}\omega_{2,mn} \\ \gamma_{3,mn}\omega_{3,mn} \end{Bmatrix} = \frac{(\mu/h)}{(I/h)}\begin{Bmatrix} 1 \\ \frac{1}{2}\left[\frac{1}{(I/h)}(k_{22,mn}+k_{33,mn}) + \frac{2}{(Ph)}k_{11,mn}\right] \\ \frac{1}{2}\frac{1}{(Ph)(I/h)}[D_{mn,22}(0)+D_{mn,33}(0)] \end{Bmatrix} - 4(\gamma_{1,mn}\omega_{1,mn})(\gamma_{2,mn}\omega_{2,mn})(\gamma_{3,mn}\omega_{3,mn})\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}, \tag{29}$$

where

$$\left.\begin{aligned}
a_{\omega 0,mn} &= \frac{1}{(Ph)(I/h)^2}[k_{11,mn}D_{mn,11}(0) + k_{12,mn}D_{mn,12}(0) + k_{13,mn}D_{mn,13}(0)] \\
a_{\omega 1,mn} &= \frac{1}{(I/h)^2}D_{mn,11}(0) + \frac{1}{(Ph)(I/h)}[D_{mn,22}(0)+D_{mn,33}(0)] + \frac{(\mu/h)^2}{(Ph)(I/h)^2}k_{11,mn} \\
&\quad -4\left[(\gamma_{1,mn}\omega_{1,mn})(\gamma_{2,mn}\omega_{2,mn})\omega_{3,mn}^2 + (\gamma_{2,mn}\omega_{2,mn})(\gamma_{3,mn}\omega_{3,mn})\omega_{1,mn}^2 + (\gamma_{3,mn}\omega_{3,mn})(\gamma_{1,mn}\omega_{1,mn})\omega_{2,mn}^2\right] \\
a_{\omega 2,mn} &= \frac{1}{(I/h)}(k_{22,mn}+k_{33,mn}) + \frac{1}{(Ph)}k_{11,mn} + \frac{(\mu/h)^2}{(I/h)^2} \\
&\quad -4\left[(\gamma_{1,mn}\omega_{1,mn})(\gamma_{2,mn}\omega_{2,mn}) + (\gamma_{2,mn}\omega_{2,mn})(\gamma_{3,mn}\omega_{3,mn}) + (\gamma_{3,mn}\omega_{3,mn})(\gamma_{1,mn}\omega_{1,mn})\right] \\
q_{\omega,mn} &= \frac{1}{3}a_{\omega 2,mn}^2 - a_{\omega 1,mn} \\
r_{\omega,mn} &= a_{\omega 0,mn} - \frac{1}{3}a_{\omega 1,mn}a_{\omega 2,mn} + \frac{2}{27}a_{\omega 2,mn}^2 \\
\theta_{\omega,mn} &= \cos^{-1}\left(-\frac{r_{\omega,mn}/2}{\sqrt{q_{\omega,mn}^3/27}}\right)
\end{aligned}\right\}. \tag{30}$$

Note that $\omega_{1,mn}$ and $\gamma_{1,mn}$ mean the undamped natural frequency and the damping factor for deflection and that $\omega_{i,mn}$ and $\gamma_{i,mn}$ ($i = 2, 3$) mean those for rotations. For the case with rotary inertia disregarded ($I = 0$), $(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}_{mn})^{-1}$ is expressed as

$$(s^2\mathbf{M}+s\mathbf{C}+\mathbf{K}_{mn})^{-1}=\frac{1}{(Ph)D_{mn,11}(0)(1+\eta_{mn})\left(\eta_{1,mn}\frac{s}{\omega_{0,mn}}+1\right)\left(\eta_{2,mn}\frac{s}{\omega_{0,mn}}+1\right)[(s+\gamma_{0,mn}\omega_{0,mn})^2+(1-\gamma_{0,mn}^2)\omega_{0,mn}^2]}\cdot\begin{bmatrix}D'_{mn,11}(s) & D'_{mn,12}(s) & D'_{mn,13}(s)\\ & D'_{mn,22}(s) & D'_{mn,23}(s)\\ \text{sym.} & & D'_{mn,33}(s)\end{bmatrix}, \tag{31}$$

where $D'_{mn,ij}(s)$ is obtained by $I=0$ in $D_{mn,ij}(s)$. $\omega_{0,mn}$ denotes the undamped natural frequency and $\gamma_{0,mn}$, η_{mn}, $\eta_{1,mn}$ and $\eta_{2,mn}$ are the damping factors, all of which are determined by the iterative method to solve the following equations:

$$\left.\begin{aligned}
&\omega_{0,mn}^2=\frac{1}{(1+\eta_{mn})}\frac{1}{(Ph)D_{mn,11}(0)}[k_{11,mn}D_{mn,11}(0)+k_{12,mn}D_{mn,12}(0)+k_{13,mn}D_{mn,13}(0)]\\
&\eta_{1,mn}\eta_{2,mn}=\frac{1}{(1+\eta_{mn})}\frac{1}{D_{mn,11}(0)}\omega_{0,mn}^2(\mu/h)^2\\
&\begin{bmatrix}\eta_{1,mn}\eta_{2,mn} & 1\\ 1 & 1\end{bmatrix}\begin{Bmatrix}2\gamma_{0,mn}\\ \eta_{1,mn}+\eta_{2,mn}\end{Bmatrix}=\frac{1}{(1+\eta_{mn})}\frac{1}{D_{mn,11}(0)}(\mu/h)\begin{Bmatrix}\omega_{0,mn}(k_{22,mn}+k_{33,mn})\\ \frac{1}{(Ph)}\frac{1}{\omega_{0,mn}}[D_{mn,22}(0)+D_{mn,33}(0)]\end{Bmatrix}\\
&\eta_{mn}=\frac{1}{1+\eta_{1,mn}\eta_{2,mn}+2\gamma_{0,mn}(\eta_{1,mn}+\eta_{2,mn})}\cdot\left\{\frac{k_{11,mn}}{(Ph)D_{mn,11}(0)}(\mu/h)^2-[\eta_{1,mn}\eta_{2,mn}+2\gamma_{0,mn}(\eta_{1,mn}+\eta_{2,mn})]\right\}
\end{aligned}\right\}. \tag{32}$$

Note that $\omega_{0,mn}$ and $\gamma_{0,mn}$ mean the undamped natural frequency and the damping factor for deflection. By substituting Eq. (26) or (31) into Eq. (25) and applying the inverse transform to Eq. (25), we obtain solutions of transient deflection and rotations due to loads varying arbitrarily with time as

$$\mathbf{x}_{mn}(t)=\int_0^t\mathbf{g}_{mn}(\tau)\mathbf{p}_{mn}(t-\tau)\,\mathrm{d}\tau+\dot{\mathbf{g}}_{mn}(t)\mathbf{M}\mathbf{x}_{mn}(0)+\mathbf{g}_{mn}(t)[\mathbf{M}\dot{\mathbf{x}}_{mn}(0)+\mathbf{C}\mathbf{x}_{mn}(0)], \tag{33}$$

where $\mathbf{g}_{mn}(t)$ denotes the weight function. $\mathbf{g}_{mn}(t)$ has different expressions for the case with rotary inertia taken into account ($I\neq 0$) and for the case with rotary inertia disregarded ($I=0$). For the case with rotary inertia taken into account ($I\neq 0$),

$$\mathbf{g}_{mn}(t)=\sum_{j=1}^{3}\mathrm{Im}\left\{\mathrm{e}^{s_{j,mn}t}(G_{j,mn}+\mathrm{i}H_{j,mn})\cdot\begin{bmatrix}D_{mn,11}(s_{j,mn}) & D_{mn,12}(s_{j,mn}) & D_{mn,13}(s_{j,mn})\\ & D_{mn,22}(s_{j,mn}) & D_{mn,23}(s_{j,mn})\\ \text{sym.} & & D_{mn,33}(s_{j,mn})\end{bmatrix}\right\}, \tag{34}$$

and for the case with rotary inertia disregarded ($I = 0$),

$$\mathbf{g}_{mn}(t) = \mathrm{Im}\left\{ \mathrm{e}^{s_{0,mn}t}(G_{0,mn} + \mathrm{i}H_{0,mn}) \cdot \begin{bmatrix} D'_{mn,11}(s_{0,mn}) & D'_{mn,12}(s_{0,mn}) & D'_{mn,13}(s_{0,mn}) \\ & D'_{mn,22}(s_{0,mn}) & D'_{mn,23}(s_{0,mn}) \\ \text{sym.} & & D'_{mn,33}(s_{0,mn}) \end{bmatrix} \right\}$$
$$+ \sum_{j=1}^{2} \exp\left(-\frac{\omega_{0,mn}}{\eta_{j,mn}}t\right) I_{j,mn} \cdot \begin{bmatrix} D'_{mn,11}\left(-\frac{\omega_{0,mn}}{\eta_{j,mn}}\right) & D'_{mn,12}\left(-\frac{\omega_{0,mn}}{\eta_{j,mn}}\right) & D'_{mn,13}\left(-\frac{\omega_{0,mn}}{\eta_{j,mn}}\right) \\ & D'_{mn,22}\left(-\frac{\omega_{0,mn}}{\eta_{j,mn}}\right) & D'_{mn,23}\left(-\frac{\omega_{0,mn}}{\eta_{j,mn}}\right) \\ \text{sym.} & & D'_{mn,33}\left(-\frac{\omega_{0,mn}}{\eta_{j,mn}}\right) \end{bmatrix}, \tag{35}$$

where

$$\left.\begin{aligned}
&s_{j,mn} = \left(-\gamma_{j,mn} + \mathrm{i}\sqrt{1-\gamma_{j,mn}^2}\right)\omega_{j,mn} \quad (j = 0,1,2,3) \\
&G_{i,mn} = \frac{1}{\sqrt{1-\gamma_{i,mn}^2}\,\omega_{i,mn}} \cdot \left\{ \frac{1}{(Ph)(I/h)^2 \Delta_{ij,mn}\Delta_{ik,mn}} [(\omega_{i,mn}^2 - \omega_{j,mn}^2)(\omega_{i,mn}^2 - \omega_{k,mn}^2) - \omega_{i,mn}^2 \delta_{ij,mn}\delta_{ik,mn}] \right. \\
&\qquad \left. + (\gamma_{i,mn}\omega_{i,mn})H_{i,mn} \right\} \quad (i,j,k = 1,2,3;\ i \neq j,\ j \neq k,\ k \neq i) \\
&H_{i,mn} = \frac{1}{(Ph)(I/h)^2 \Delta_{ij,mn}\Delta_{ik,mn}} \cdot \left[(\omega_{i,mn}^2 - \omega_{j,mn}^2)\delta_{ik,mn} + (\omega_{i,mn}^2 - \omega_{k,mn}^2)\delta_{ij,mn} - 2(\gamma_{i,mn}\omega_{i,mn})\delta_{ij,mn}\delta_{ik,mn} \right] \\
&\qquad (i,j,k = 1,2,3;\ i \neq j,\ j \neq k,\ k \neq i) \\
&\Delta_{ij,mn} = \left(\omega_{i,mn}^2 - \omega_{j,mn}^2\right)^2 - \delta_{ij,mn}\left[2(\gamma_{i,mn}\omega_{i,mn})(\omega_{i,mn}^2 - \omega_{j,mn}^2) - \omega_{i,mn}^2\delta_{ij,mn}\right] \quad (i,j = 1,2,3) \\
&\delta_{ij,mn} = 2(\gamma_{i,mn}\omega_{i,mn} - \gamma_{j,mn}\omega_{j,mn}) \quad (i,j = 1,2,3) \\
&G_{0,mn} = \frac{1}{(Ph)D_{mn,11}(0)\sqrt{1-\gamma_{0,mn}^2}\,\omega_{0,mn}(1+\eta_{mn})} \frac{1}{\Delta_{mn}} \cdot [1 - \gamma_{0,mn}(\eta_{1,mn} + \eta_{2,mn}) - (1 - 2\gamma_{0,mn}^2)\eta_{1,mn}\eta_{2,mn}] \\
&H_{0,mn} = \frac{1}{(Ph)D_{mn,11}(0)\omega_{0,mn}(1+\eta_{mn})} \frac{1}{\Delta_{mn}} [2\gamma_{0,mn}\eta_{1,mn}\eta_{2,mn} - (\eta_{1,mn} + \eta_{2,mn})]; \\
&I_{i,mn} = \frac{1}{(Ph)D_{mn,11}(0)\omega_{0,mn}(1+\eta_{mn})} \frac{\eta_{i,mn}^2}{\eta_{i,mn} - \eta_{j,mn}} \frac{1}{\Delta_{mn}} [1 + \eta_{j,mn}(\eta_{j,mn} - 2\gamma_{0,mn})] \quad (i,j = 1,2; i \neq j) \\
&\Delta_{mn} = 1 - 2\gamma_{0,mn}(\eta_{1,mn} + \eta_{2,mn}) + (\eta_{1,mn} + \eta_{2,mn})^2 \\
&\qquad -2\eta_{1,mn}\eta_{2,mn}(1 - 2\gamma_{0,mn}^2) - 2\gamma_{0,mn}\eta_{1,mn}\eta_{2,mn}(\eta_{1,mn} + \eta_{2,mn}) + (\eta_{1,mn}\eta_{2,mn})^2
\end{aligned}\right\}. \tag{36}$$

Thus, transient deflection and rotations are expressed by the summation of the terms due to loads varying dynamically with time and the terms due to the initial conditions. Moreover, the terms due to loads varying dynamically with time are expressed by the convolution of the function representing the dynamic loads and the weight function which is determined independently of the dynamical loads.

The solution based on the CLT where the effect of transverse shear is disregarded are shown. In the CLT, the rotations ψ_x and ψ_y are expressed by

$$\psi_x = -\frac{\partial w}{\partial x}, \quad \psi_y = -\frac{\partial w}{\partial y}. \tag{37}$$

As the equation of motion, the equation which is obtained by eliminating Q_x and Q_y in Eq. (5) is used. Solution of transient deflection under the CLT is also solved in the same manner as that under the FSDT and is obtained as

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}(t) \sin \alpha_m x \sin \beta_n y \tag{38}$$

and

$$\frac{w_{mn}(t)}{h} = \int_0^t g_{c,mn}(\tau) p_{c,mn}(t-\tau)\,\mathrm{d}\tau + \dot{g}_{c,mn}(t) M_c \frac{w_{mn}(0)}{h} + g_{c,mn}(t)\left[M_c \frac{\dot{w}_{mn}(0)}{h} + C_c \frac{w_{mn}(0)}{h}\right], \tag{39}$$

where

$$\left.\begin{aligned}
&g_{c,mn}(t) = \frac{1}{M_c\sqrt{1-\gamma_{c,mn}^2}\,\omega_{c,mn}} \mathrm{e}^{-\gamma_{c,mn}\omega_{c,mn}t} \sin\left(\sqrt{1-\gamma_{c,mn}^2}\,\omega_{c,mn} t\right)\\
&\omega_{c,mn}^2 = \frac{K_{c,mn}}{M_c}, \quad \gamma_{c,mn} = \frac{C_c}{2\sqrt{M_c K_{c,mn}}}\\
&M_c = (Ph) + (I/h)[(h\alpha_m)^2 + (h\beta_n)^2], \quad C_c = (\mu/h)[(h\alpha_m)^2 + (h\beta_n)^2]\\
&K_{c,mn} = h[D_{11}\alpha_m^4 + D_{22}\beta_n^4 + 2(D_{12} + 2D_{66})\alpha_m^2\beta_n^2]\\
&p_{c,mn}(t) = q_{mn}(t) + \alpha_m^2 H_{mn}(t) + \beta_n^2 I_{mn}(t) + \frac{1}{2}(\bar{e}_{31}\alpha_m^2 + \bar{e}_{32}\beta_n^2)(z_k + z_{k-1})[V_{mn}^k(t) + V_{mn}^{k'}(t)]
\end{aligned}\right\}. \tag{40}$$

3. Numerical calculation

Some numerical calculation is carried out in order to examine the effect of damping due to interlaminar shear and the effect of transverse shear on the dynamic behavior of the laminate and how the voltage applied to the laminate decreases the deflection due to unavoidable thermal load.

We assume that the piezoelectric layers are of $BaTiO_3$ and other layers are of graphite/epoxy (GE). Reduced material constants are given as follow:

for GE layer (Tauchert, 1992; Tang and Xu, 1995);

$$\left.\begin{aligned}
&Q_{11}^e = 182\ \text{GPa}, \quad Q_{22}^e = 10.3\ \text{GPa}, \quad Q_{12}^e = 2.90\ \text{GPa}\\
&Q_{44}^e = 2.87\ \text{GPa}, \quad Q_{55}^e = 7.17\ \text{GPa}, \quad Q_{66}^e = 7.17\ \text{GPa}\\
&\lambda_1^e = 68.8\times 10^3\ \text{Pa}\,\text{K}^{-1}, \quad \lambda_2^e = 233\times 10^3\ \text{Pa}\,\text{K}^{-1}\\
&\rho^e = 1.580\times 10^3\ \text{kg}\,\text{m}^{-3}
\end{aligned}\right\}, \tag{41}$$

and for $BaTiO_3$ (which exhibits 6 mm symmetry) layer (Dunn, 1993; Wang and Yu, 2000);

$$\left.\begin{array}{l} Q^p_{11} = Q^p_{22} = 120 \text{ GPa}, \quad Q^p_{12} = 36.2 \text{ GPa}, \quad Q^p_{44} = Q^p_{55} = 44.0 \text{ GPa}, \quad Q^p_{66} = 42.0 \text{ GPa} \\ \lambda^p_1 = \lambda^p_2 = 1.33 \times 10^6 \text{ Pa K}^{-1}, \\ e_{31} = e_{32} = -12.3 \text{ C m}^{-2}, \quad e_{15} = e_{24} = 11.4 \text{ C m}^{-2} \\ \eta^p_{11} = \eta^p_{22} = 9.87 \times 10^{-9} \text{ C}^2\text{N}^{-1}\text{m}^{-2}, \quad \eta^p_{33} = 13.2 \times 10^{-9} \text{ C}^2\text{N}^{-1}\text{m}^{-2} \\ \rho^p = 5.700 \times 10^3 \text{ kg m}^{-3} \end{array}\right\}, \tag{42}$$

where η^p_{ij} denotes the permittivities of $BaTiO_3$. Reduced and transformed material properties can be obtained according to the work by Jonnalagadda et al. (1994). The damping coefficient of the laminate, μ, is considered to depend on the way to unite the laminae practically. Here we assume the non-dimensional parameter as

$$\frac{\mu}{\sqrt{Q^e_{11}\rho^e h^4}} = 0.2 \tag{43}$$

otherwise stated. We assume that the square layers ($a = b$) are piled as $\{[BaTiO_3{:}0°]/[GE{:}(90°/0°)_2]\}_{sym.}$ ($N = 10$, $k = 1$, $k' = 10$) and that each layers has the same thickness. Parameters k_1 and k_2 in Eq. (12) are taken to be unit.

Hereafter, we use quantities

$$\left.\begin{array}{ll} \Omega_{i,mn} = \sqrt{1-\gamma^2_{i,mn}}\,\omega_{i,mn}, & \delta_{i,mn} = \gamma_{i,mn}\omega_{i,mn} \\ \Omega^{CLT}_{i,mn} = \sqrt{1-\gamma^2_{c,mn}}\,\omega_{c,mn}, & \delta^{CLT}_{i,mn} = \gamma_{c,mn}\omega_{c,mn} \end{array}\right\} \quad (i = 0, 1), \tag{44}$$

where Ω and δ are referred to as the damped natural frequency and the decay rate, respectively, for deflection and it should be noted that subscripts $i = 0$ and 1 denote the results for the case with rotary inertia disregarded ($I = 0$) and for the case with rotary inertia taken into account ($I \neq 0$), respectively, and that superscript CLT denotes the results under CLT.

3.1. The effects of transverse shear and damping on the natural frequencies and the decay rates

Fig. 3 shows the variation of the ratio of the minimum damped natural frequency and the decay rate for deflection based on the FSDT to those based on the CLT with the length-to-thickness ratio a/h (referred to as 'LT ratio') for $\mu/\sqrt{Q^e_{11}\rho^e h^4} = 0.2$ where the rotary inertia is disregarded. From Fig. 3, it is found that the

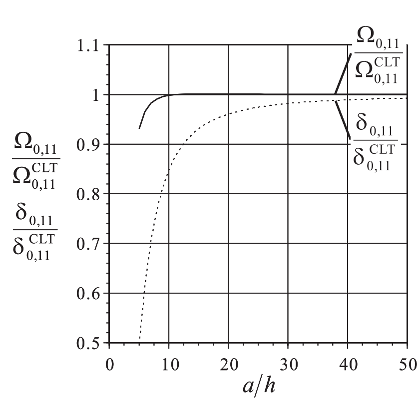

Fig. 3. Variation of minimum damped natural frequency and decay rate with length-to-thickness ratio ($\mu/\sqrt{Q^e_{11}\rho^e h^4} = 0.2$).

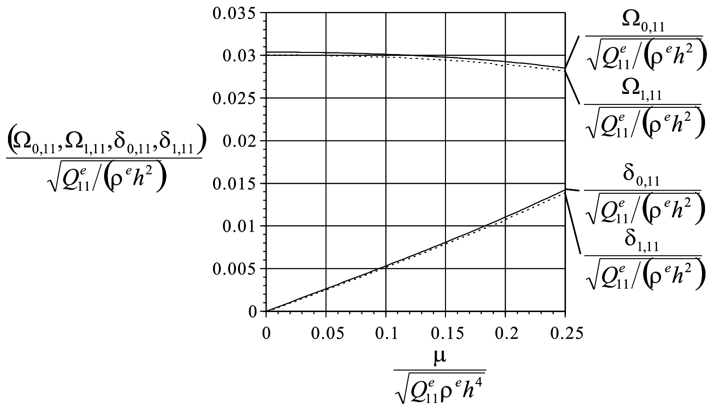

Fig. 4. Variation of minimum damped natural frequencies and decay rates with damping coeffcient ($a/h = 10$).

effect of transverse shear on the minimum damped natural frequency and the decay rate gets more significant as the LT ratio gets small. Moreover, it is found that the minimum natural frequency based on the FSDT $\Omega_{0,11}$ is smaller than that based on the CLT $\Omega_{0,11}^{\mathrm{CLT}}$ and that the decay rate based on the FSDT $\delta_{0,11}$ is smaller than that based on the CLT $\delta_{0,11}^{\mathrm{CLT}}$.

Fig. 4 shows the variation of the minimum damped natural frequencies and the decay rates for deflection with the damping coefficient of the laminate under the FSDT. From Fig. 4, it is found that the minimum natural frequencies decreases and the decay rate increases as the damping coefficient increases. Moreover, significant difference between the minimum natural frequency with rotary inertia disregarded ($\Omega_{0,11}$) and that with rotary inertia taken into account ($\Omega_{1,11}$) is not found. Also significant difference between the decay rate with rotary inertia disregarded ($\delta_{0,11}$) and that with rotary inertia taken into account ($\delta_{1,11}$) is not found.

3.2. The effects of transverse shear and damping on transient deflection

The effects of transverse shear and damping on the transient deflection due to thermal load are investigated. Hereafter, the rotary inertia are assumed to be neglected ($I = 0$). In this section, we consider that the laminate is still at its undeformed state initially and that it is subjected to a sudden temperature change and is kept at the temperature on the lower surface as an unavoidable thermal environment:

$$\left.\begin{array}{l} T_{N,11}(t) - T_{0,11}(t) = T_0 H(t), \quad T_{N,mn}(t) - T_{0,mn}(t) = 0 \quad (m,n) \neq (1,1) \\ V^k_{mn}(t) = V^{k'}_{mn}(t) = 0 \quad \text{for all } (m,n) \\ q_{mn}(t) = 0 \quad \text{for all } (m,n) \end{array}\right\}, \tag{45}$$

where $H(t)$ denotes the Heaviside unit function. Then, these loads cause $w_{mn}(t)$ only for $(m,n) = (1,1)$.

Fig. 5 shows the transient behavior of the thermal deflection at the center of the laminate under the FSDT. From Fig. 5, it is found that the thermal deflection vibrates and decays and that it tends to the final deflection, which agrees with the thermal deflection under the static analysis by us (Ishihara and Noda, 2000). Hereafter, the final deflection is denoted by $\langle w \rangle$. Moreover, it is found that the thermal deflection decays more rapidly and the frequency gets smaller for the larger value of the damping coefficient, which corresponds to the results in Fig. 4.

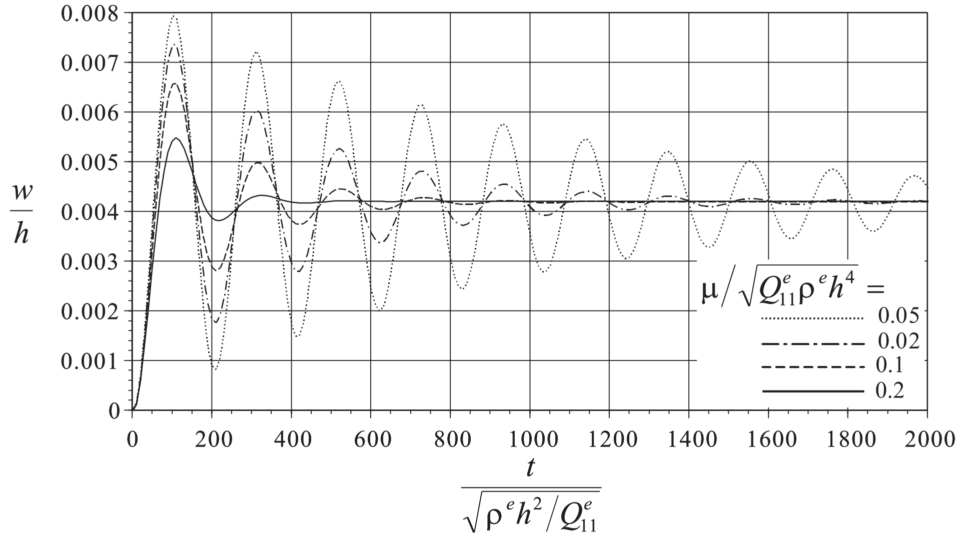

Fig. 5. Transient deflection due to sudden temperature change ($a/h = 10$).

3.3. Control of thermal deflection by applied voltage

We treat control of thermal deflection by applying the electrical voltage to the piezoelectric layers. Let $w_{\text{uncontrol}}$ denote the deflection due to unavoidable thermal load and $w_{\text{electrical}}$ denote the deflection due to the electrical voltage to compensate the effect of the thermal load. Then, the total deflection w_{control} is expressed by

$$w_{\text{control}} = w_{\text{uncontrol}} + w_{\text{electrical}}. \tag{46}$$

As the first example, we assume that the laminate which is still at its undeformed state initially is subjected to the sudden temperature change described by Eq. (45), that it reaches to the final deflection at t_∞ and that then the sustaining electrical voltage starts to be applied to the upper surface of the kth piezoelectric layer as

$$\left.\begin{aligned} &V^k_{11}(t) = -VH(t - t_\infty),\ V^k_{mn}(t) = 0 \quad (m,n) \neq (1,1) \\ &V^{k'}_{mn}(t) = 0 \quad \text{for all } (m,n) \end{aligned}\right\}. \tag{47}$$

Fig. 6 shows the corresponding transient behavior of deflection w_{control} where t_∞ is taken as infinity. From Fig. 6, it is found that the thermal deflection is suppressed by the sustaining electrical voltage applied to the piezoelectric layer and that the final deflection is linear with respect to the magnitude of the applied electrical voltage.

As the second example, we assume that the laminate which is still at its undeformed state initially is subjected to impulsive thermal load as

$$\left.\begin{aligned} &T_{N,11}(t) - T_{0,11}(t) = T_0 \delta\left(\frac{t}{\sqrt{\rho^e h^2/Q^e_{11}}}\right), \quad T_{N,mn}(t) - T_{0,mn}(t) = 0 \quad (m,n) \neq (1,1) \\ &q_{mn}(t) = 0 \text{ for all } (m,n) \end{aligned}\right\}, \tag{48}$$

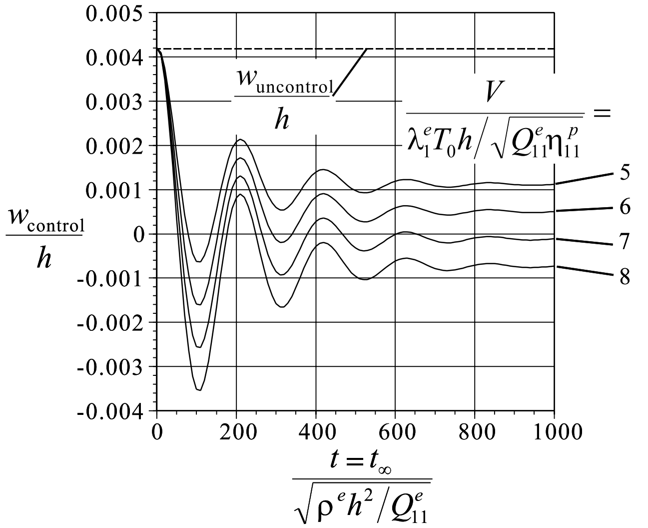

Fig. 6. Control of thermal deflection by sustaining electrical voltage ($a/h = 10$, $\mu/\sqrt{Q_{11}^e \rho^e h^4} = 0.1$).

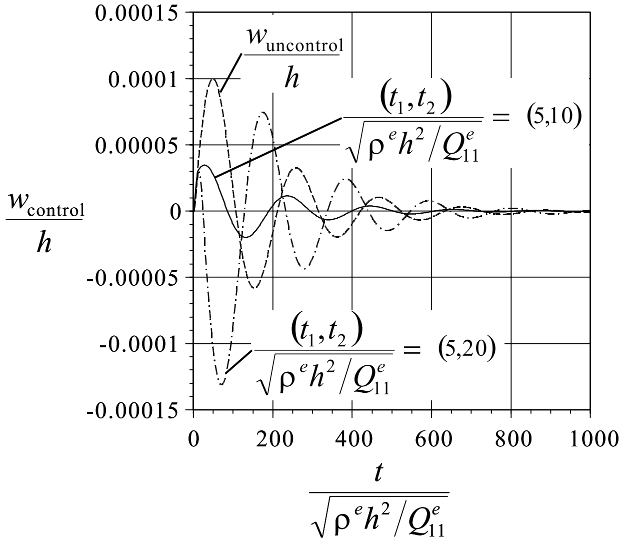

Fig. 7. Control of the thermal deflection due to impulsive thermal load by a pulse of electrical voltage ($a/h = 10$, $\mu/\sqrt{Q_{11}^e \rho^e h^4} = 0.1$, $V/(\lambda_1^e T_0 h/\sqrt{Q_{11}^e \eta_{11}^p}) = 1$).

and that then it is subjected to a pulse of electrical voltage during t_1 to t_2 ($>t_1$) as

$$\left.\begin{array}{l} V_{11}^k(t) = V[H(t - t_1) - H(t - t_2)], \quad V_{mn}^k(t) = 0 \quad (m, n) \neq (1, 1) \\ V_{mn}^{k'}(t) = 0 \text{ for all } (m, n) \end{array}\right\} \tag{49}$$

on the upper surface of the kth piezoelectric layer. Fig. 7 shows the transient behavior of deflection w_{control} for some combinations of t_1 and t_2. From Fig. 7, it is found that the deflection is suppressed for

$(t_1, t_2)/\sqrt{\rho^e h^2/Q^e_{11}} = (5, 10)$ and is increased for $(t_1, t_2)/\sqrt{\rho^e h^2/Q^e_{11}} = (5, 20)$. Thus, the deflection due to impusive thermal load is found to be suppressed by appropriate duration of the pulse of electrical voltage to the piezoelectric layer. Next, we define the suppression rate R as

$$R = \frac{|w_{\text{control}}|_{\text{maximum}}}{|w_{\text{uncontrol}}|_{\text{maximum}}}. \tag{50}$$

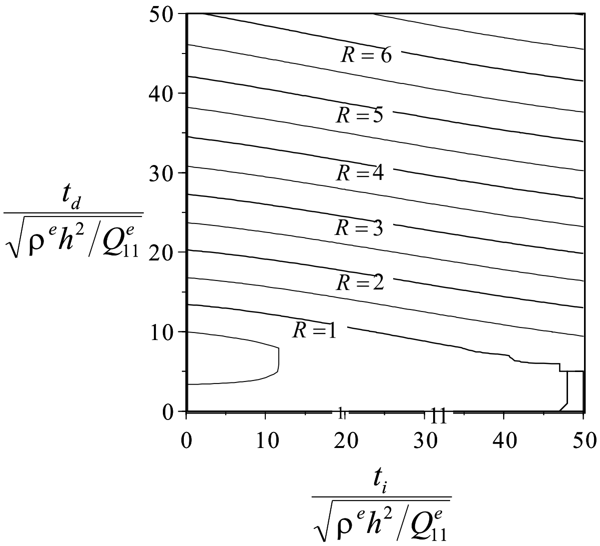

Fig. 8. Variation of the suppression rate with the initial time and the duration of the pulse of electrical voltage ($a/h = 10$, $\mu/\sqrt{Q^e_{11}\rho^e h^4} = 0.1$, $V/(\lambda^e_1 T_0 h/\sqrt{Q^e_{11}\eta^p_{11}}) = 1$).

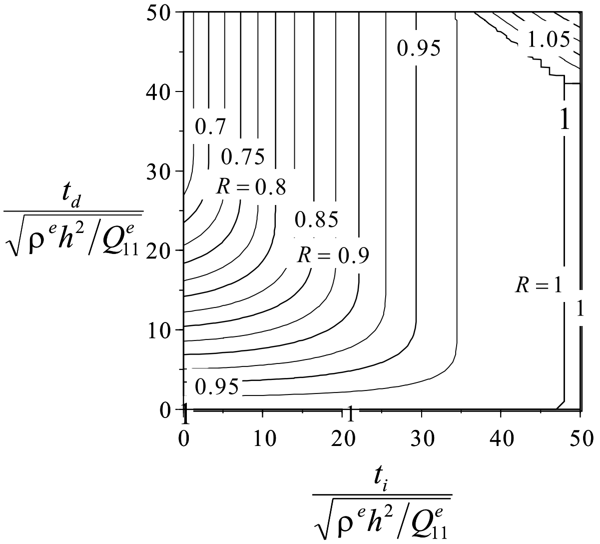

Fig. 9. Variation of the suppression rate with the initial time and the duration of the pulse of electrical voltage ($a/h = 10$, $\mu/\sqrt{Q^e_{11}\rho^e h^4} = 0.1$, $V/(\lambda^e_1 T_0 h/\sqrt{Q^e_{11}\eta^p_{11}}) = 0.1$).

Figs. 8 and 9 show the variation of the suppression rate with the initial time t_i $(=t_1)$ and the duration t_d $(=t_2 - t_1)$ of the pulse of electrical voltage. From Fig. 8, it is found that, in order to suppress the maximum deflecion ($R < 1$) using relatively large magnitude of the voltage $(V/(\lambda_1^e T_0 h/\sqrt{Q_{11}^e \eta_{11}^p}) = 1)$, the initial time must be earlier than some specific time $(t_i/\sqrt{\rho^e h^2/Q_{11}^e} < 43)$ and the duration must be within some period depending on the initial time. From Fig. 9, it is found that, in order to suppress the maximum deflecion for relatively small magnitude of the voltage $(V/(\lambda_1^e T_0 h/\sqrt{Q_{11}^e \eta_{11}^p}) = 0.1)$, the initial time must be earlier than some specific time and the duration can range wider than the case shown in Fig. 8. Moreover, from Fig. 9, it is found that, for relatively small magnitude of the voltage, the pulse with the earlier initial time and the longer duration is able to suppress the maximum deflection more.

4. Concluding remarks

We study dynamic behavior of a piezothermoelastic laminate considering the effect of damping due to interlaminar shear and the effect of transverse shear and obtain the solution of transient response. Moreover, from numerical calculation, the following remarks are found:

1. The effect of transverse shear on the minimum damped natural frequency and the decay rate gets more significant as the length-to-thickness ratio decreases.
2. The minimum damped natural frequency decreases and the decay rate increases by the damping due to interlaminar shear.
3. The final thermal deflection due to a sudden and sustaining temperature change can be suppressed by applying sustaining electrical voltage to the piezoelectric actuator.
4. The transient thermal deflection due to impulsive thermal load can be suppressed by appropriate pulse of electrical voltage to the piezoelectric actuator.
5. The maximum deflection due to impulsive thermal load can be suppressed more effectively by the pulse of electrical voltage with relatively small magnitude, the earlier initial time and the longer duration.

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